ELSEVIER Journal of Constructional Steel Research 52 (1999) 159–170

JOURNAL OF CONSTRUCTIONAL STEEL RESEARCH

www.elsevier.com/locate/jcsr

Stress concentration factor in plates with transverse butt-weld misalignment

Weicheng Cui[a,*], Zhengquan Wan[b], Alaa E. Mansour[c]

[a]*School of Naval Architecture and Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200030, People's Republic of China*

[b]*China Ship Scientific Research Center, P.O. Box 116, Wuxi, Jiangsu 214082, People's Republic of China*

[c]*Department of Naval Architecture and Offshore Engineering, University of California at Berkeley, Berkeley, CA 94720, USA*

Received 21 July 1998; accepted 15 April 1999

Abstract

Misalignment at butt-welded joints induces bending stresses with the application of in-plane loads only. This stress concentration will have a detrimental effect on both the ultimate strength of the plate and the fatigue strength at the weld. The stress concentration factor is often represented as $K = 1 + C(e/t)$, where C is called the stress concentration coefficient in this paper. In the current literature it is found that different values of C have been used, none of which were derived by the plate theory. In this paper, a linear elastic stress analysis of ship plates with a tranverse butt-weld misalignment under uniaxial compression is carried out. It is found that the stress concentration coefficient C is not a constant, but varies with the aspect ratio, the location of the misalignment in the plate and the magnitude of the nominal applied stress.

Keywords: Plate; Misalignment; Stress concentration factor; Uniaxial compression

Nomenclature

a	plate length
b	plate width
C	stress concentration coefficient anywhere in the plate

* Corresponding author.

0143-974X/99/$ - see front matter
PII: S0143-974X(99)00019-X

C_e	stress concentration coefficient at misalignment
D	$Et^3/12(1 - \nu^2)$
E	Young's modulus
e	misalignment
K	stress concentration factor anywhere in the plate
K_e	stress concentration factor at misalignment
L_1, L_2	lengths of plate 1 and plate 2, respectively
N_{cr}	Euler buckling load of the plate strip
N_{xn}	nominal load of the plate which is used for normalization
R	relative location of the misalignment ($= L_1/a$)
t	plate thickness
w	deflection
x_i, y_i	location coordinates
α	aspect ratio ($= a/b$)
ν	Poisson's ratio
ϕ_x	normalized applied load

1. Introduction

Unstiffened plates are the main structural components in ship structure. The buckling and ultimate strengths of these elements are very important from the design and safety viewpoint because the collapse loads of these elements can often act as an indicator of the ultimate strength of the whole stiffened panel [1]. Therefore, this problem has received very wide attention (e.g., [1–5]). The factors which might affect the buckling and ultimate strengths of an unstiffened plate include:

1. the type of loading, such as uniaxial compression, in-plane bending, in-plane shear, biaxial compression, lateral pressure, etc. The most typical loading for an unstiffened plate in a ship structure is the combination of uniaxial compression and lateral pressure;
2. the boundary conditions, such as simply supported and clamped. In actual ship structures, the longitudinal and tranverse stiffeners will always have limited but not zero stiffness. Therefore, the actual boundary conditions will always be somewhere between simply supported and fully clamped. There are two types of restraints in the boundary conditions [6]. One is the restraint against lateral slide and the other is against rotation. These two parameters will also affect the buckling and ultimate strengths;
3. the plate material properties and geometric dimensions, such as aspect ratio and slenderness;
4. plate unfairness and residual stresses, which are caused by welding of the stiffener attachment to the plating. Both distortion shape and amplitude of the unfairness will affect the plate strength [2–5]; and
5. misalignment, which is induced by butt welding to join plates together [7–12]. When fabricating a large complex structure like a ship, using butt welding to join

plates together is a common process. No matter how careful the fabrication process is, misalignment between the two jointed plates always exists. The most general form of misalignment should include both angular mismatch and eccentricity (or offset), see Fig. 1. In this paper, only the eccentricity type of misalignment is considered.

However, most of the current literature concentrates only on the problems of the first four factors and pays particular attention to the effects of initial distortion and residual stresses, while few works are dedicated to the misalignment [7–12]. Misalignment at butt-welded joints will induce bending stresses with the application of in-plane loads only. This stress concentration factor is often represented as $K = 1 + C_e(e/t)$, where C_e is called the stress concentration coefficient at misalignment in this paper. Based on experimental results, Gunn and McLester [8] recommended $C_e = 3.0$. This result has been widely used [9,13]. Refs. [10,11] attempted to derive the coefficient from theory, but they simplified the problem by using columns or frames instead of plates. In comparison with columns, the effect of boundary constraints in plates is expected to be higher. In the ABS classification rules for ships [14], the value of C_e has been changed to 1.5. However, no justification for this has been found.

The purpose of this paper is to carry out linear elastic stress analysis based on the small deflection theory of plates. The mathematical model explicitly includes the misalignment. The work in this paper can be viewed as an extension to Ref. [7] in which misaligned columns were analyzed.

2. Problem description

The unstiffened plate has dimensions $a \times b \times t$ (mm × mm × mm). There exists a butt weld containing a misalignment e across the whole width. The distance from the left side to the misalignment is L_1 and the distance from the right side to the misalignment is L_2 ($= a - L_1$). The plate is subjected to a uniaxial compression. Fig. 2 shows the boundary conditions and the coordinate systems. Note that the misalignment is enlarged for clear presentation.

3. Column model

If the plate is very wide in the transverse direction, namely $b \gg a$, its curvature along the y-axis will be small. In this case, we can use an elemental strip of plating

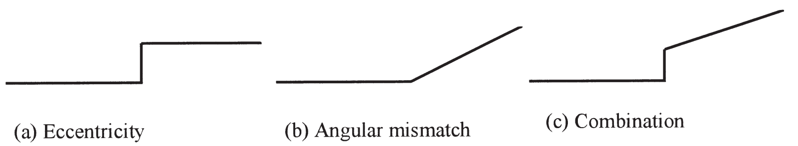

Fig. 1. Types of butt-weld misalignments.

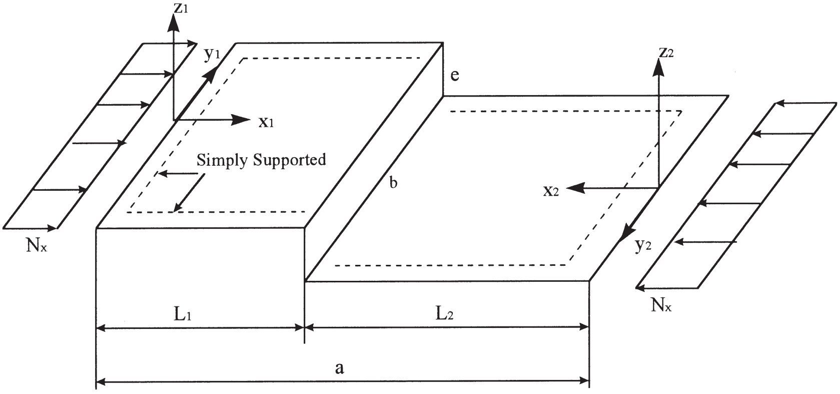

Fig. 2. Boundary conditions and coordinate systems for the misaligned plate.

of unit width in the analysis, see Fig. 3. Then the classical beam–column model can be applied.

The governing differential equations for the problem are:

$$\frac{d^4w_i}{dx_i^4} + \frac{N_x}{EI}\frac{d^2w_i}{dx_i^2} = 0 \quad (i = 1, 2). \tag{1}$$

The boundary conditions are:

$$w_i(0) = w_i''(0) = 0 \quad (i = 1, 2). \tag{2}$$

The conditions of continuity at misalignment are:

$$w_1(L_1) = w_2(L_2), \quad w_1'(L_1) = -w_2'(L_2), \quad M_{x_1}(L_1) - M_{x_2}(L_2) = N_x e, \quad Q_{x_1}(L_1) = -Q_{x_2}(L_2). \tag{3}$$

The solution to this problem can be easily derived and this has been given in [7]. For the convenience of discussion, let us introduce the following parameters for the plate strip:

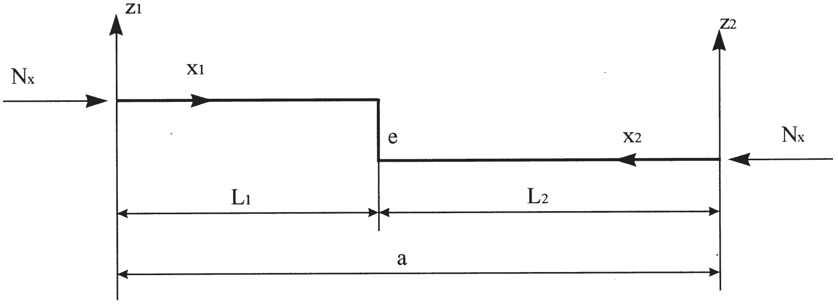

Fig. 3. Coordinate systems for the misaligned columns.

$$\phi_x = \frac{N_x}{N_{cr}}, \quad N_{cr} = \frac{\pi^2 E t^3}{12a^2}, \quad R = \frac{L_1}{a}, \quad s_0 = \sqrt{\phi_x}, \quad K_e = 1 + C_e \frac{e}{t}. \tag{4}$$

Then the coefficient of stress concentration factor at misalignment can be expressed as:

$$C_e = \mathrm{Max}\left(\frac{6}{1 + \mathrm{tg}(\pi s_0 R)\,\mathrm{ctg}[\pi s_0(1-R)]}, \frac{6}{1 + \mathrm{ctg}(\pi s_0 R)\,\mathrm{tg}[\pi s_0(1-R)]}\right). \tag{5}$$

C_e is generally a function of the relative location of misalignment R and the applied load level, ϕ_x. Fig. 4 shows the effect of R and ϕ_x on C_e. It can be seen that when the misalignment is in the middle, i.e., $R = 0.5$, then C_e is a constant which equals 3. This result corresponds to the famous formula of Eq. (6). However, when the misalignment is not in the middle, then C_e is generally greater than 3. Both R and ϕ_x have an influence on C_e. The maximum value of C_e is 6.

$$K_e = 1 + 3e/t, \tag{6}$$

Eq. (6) was first developed by Gunn and McLester [8] based on experiments. Subsequently, a similar formula was also derived theoretically by Berge and Myhre [10] but they showed that it was necessary to take into account the overall restraint imposed upon the joint. The formula is also widely used to calculate the stress concentration factor due to weld misalignment in plates [9,13]. However, from the above discussion, it can be seen that the formula is based on the column model and only valid for the case of misalignment in the middle.

4. Linear elastic stress solution of unstiffened plates with misalignment

By applying the linear elastic plate theory to solve the problem described in Section 2, the deflection can be found. The details are given in Appendix A. In this

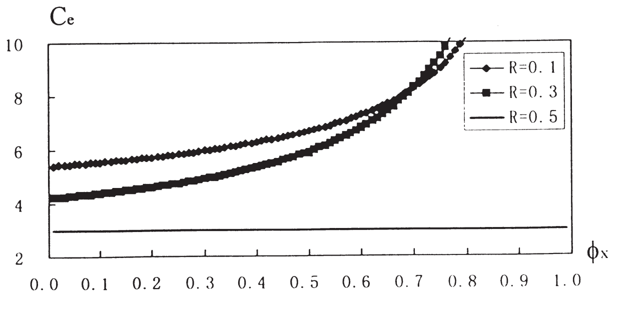

Fig. 4. Effect of applied load and the misalignment location on the stress concentration coefficient C_e.

section, only the results are presented. For convenience of discussion, we also express the results in terms of the following parameters:

$$\alpha = \frac{a}{b} = \frac{L_1 + L_2}{b}, \quad R = \frac{L_1}{a}, \quad \phi_x = \frac{N_x}{N_{xn}}, \quad N_{xn} = \frac{4\pi^2 D}{b^2}, \quad \epsilon_0 = \epsilon a = a\pi c_0, \quad \eta_0 = \eta a = a\pi s_0, \quad c_0 = \sqrt{1-\phi_x}, \quad s_0 = \sqrt{\phi_x}. \tag{7}$$

If we assume that A_{10}, B_{10}, A_{20} and B_{20} are the solutions of the following equation:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \begin{Bmatrix} A_{10} \\ B_{10} \\ A_{20} \\ B_{20} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix}, \tag{8}$$

and denote the stress concentration factor as:

$$K = 1 + C\frac{e}{t}, \tag{9}$$

then the coefficient C can be expressed as:

$$C(x_i, y_i) = 3\pi\{[A_{i0}(c_0^2 - s_0^2 - \nu) - 2B_{i0}c_0s_0]\cosh(\epsilon x_i)\sin(\eta x_i) + [2A_{i0}c_0s_0 + B_{i0}(c_0^2 - s_0^2 - \nu)]\sinh(\epsilon x_i)\cos(\eta x_i)\}\cos\left(\frac{\pi y_i}{b}\right) \quad (i = 1, 2). \tag{10}$$

From Eq. (10), it can be seen that C is parabolically distributed along the width direction and the maximum values occur at $y_i = 0$. Thus the maximum stress concentration at the misalignment can be expressed as:

$$C_e = \mathrm{Max}(|C_1|, |C_2|), \quad C_1 = 3\pi\{[A_{10}(c_0^2 - s_0^2 - \nu) - 2B_{10}c_0s_0]T_1 + [2A_{10}c_0s_0 + B_{10}(c_0^2 - s_0^2 - \nu)]T_2\}, \quad C_2 = 3\pi\{[A_{20}(c_0^2 - s_0^2 - \nu) - 2B_{20}c_0s_0]T_5 + [2A_{20}c_0s_0 + B_{20}(c_0^2 - s_0^2 - \nu)]T_6\}. \tag{11}$$

It can be seen that C_e is a function of α, R and ϕ_x. Fig. 5 shows the effect of applied load level ϕ_x on the stress concentration coefficient at the misalignment, C_e. When the misalignment is in the middle, the applied load level has no effect on C_e. When the misalignment is not in the middle, then C_e increases with ϕ_x; however, when $R > 0.1$ and $\phi_x < 0.5$, then C_e is practically independent of the applied load.

Fig. 6 shows the effect of R on the stress concentration coefficient C_e. It can be seen that as R is away from the middle, the stress concentration increases. However, for the range of $0.4 < R < 0.6$, C_e is almost a constant.

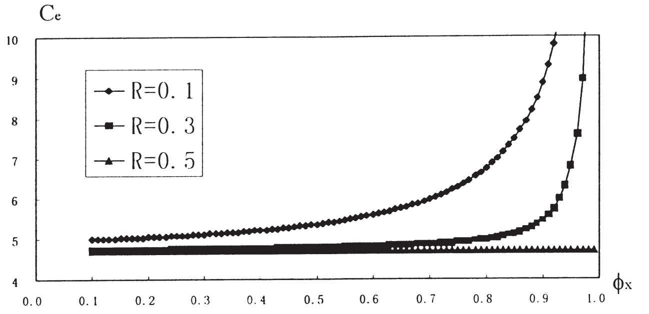

Fig. 5. Effect of applied load level ϕ_x on the stress concentration coefficient C_e ($\alpha = 3$).

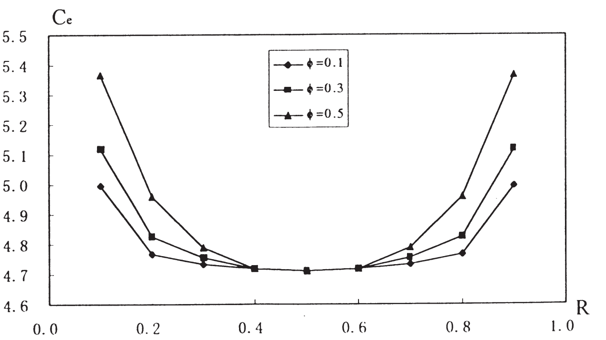

Fig. 6. Effect of the misalignment location R on the stress concentration coefficient C_e ($\alpha = 3$).

Fig. 7 shows the influence of aspect ratio α on the stress concentration coefficient C_e. It can be seen that for very long plates ($\alpha > 4.0$), C_e approaches a constant. This constant is a maximum value along the width direction. If we calculate the average along the width direction, then the mean value is 3, which is the result of Gunn and McLester's experiments [8]. For wider plates ($\alpha < 0.5$), C_e is a function of R only.

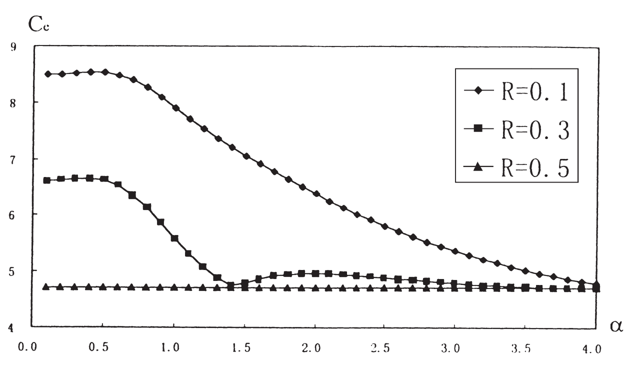

Fig. 7. Effect of aspect ratio α on the stress concentration coefficient C_e ($\phi_x = 0.5$).

If we transfer the maximum value into the mean value, then the results are the same as that from the column model.

In ABS classification rules for ships [14], the formula recommended is:

$$K_e = 1 + 1.5e/t. \tag{12}$$

From the results obtained in this analysis, it can be seen that this formula has no theoretical basis. Furthermore, it is in the lower side which is unconservative, while the formula recommended in [8] is a mean value along the width direction for plates with a misalignment in the middle range.

5. Summary and conclusions

Misalignment at butt-welded joints induces bending stresses with the application of in-plane loads only. This stress concentration will have a detrimental effect on both the ultimate strength of the plate and the fatigue strength at the weld. However, different values of the stress concentration coefficient at misalignment have been used in the literature [9,13,14], none of which were obtained from the plate theory. In this paper, a linear elastic stress analysis of ship plates with a transverse butt-weld misalignment under uniaxial compression is carried out. Through this analysis, the following conclusions can be drawn.

1. Strictly speaking, the stress concentration coefficient at misalignment C_e is not a constant, but varies with the location of the weld, the aspect ratio of the plate and the magnitude of the applied load. However, when the butt welding is within the middle range ($0.4 < R < 0.6$), then both the aspect ratio and the magnitude of the applied load have a negligible effect on C_e and C_e has the lowest value. This indicates that, to achieve a better strength of a plate with a transverse butt-weld misalignment, the butt welding must be in the middle range.
2. The stress distribution along the width direction is not uniform and the maximum value occurs in the center. The popularly used value of $C_e = 3$ corresponds to the mean value for the butt welding in the middle range or corresponds to the column model. The maximum value of C_e for these cases is 4.712.
3. The results from the plate theory are consistent with the results from the column model.

Acknowledgements

This research was carried out during the first author's visit to the University of California at Berkeley. Financial support from the Chinese Scholarship Council for the visit of the first author is highly appreciated. The support of NAVSEA contract no. N00024-96-C-4123 is also acknowledged.

Appendix A

The coordinate systems are shown in Fig. 2. The governing differential equations for the problem are:

$$\frac{\partial^4 w_i}{\partial x_i^4} + 2\frac{\partial^4 w_i}{\partial^2 x_i^2 \partial^2 y_i^2} + \frac{\partial^4 w_i}{\partial y_i^4} + \frac{N_x}{D}\frac{\partial^2 w_i}{\partial x_i^2} = 0 \quad (i = 1, 2), \tag{A1}$$

where $w_1(x_1, y_1)$ and $w_2(x_2, y_2)$ represent the deflections of plate 1 and plate 2, respectively. In this equation, N_x is positive for compression.

These two fourth-order partial differential equations require 16 boundary conditions in order to have a definite solution. These are: (1) 12 boundary conditions along the six simply supported sides; and (2) four conditions of continuity at misalignment. Thereore, the boundary conditions of this problem are:

$$\begin{aligned} &w_i\Big|_{y_i = \pm b/2} = 0, \quad \frac{\partial^2 w_i}{\partial y_i^2}\Big|_{y_i = \pm b/2} = 0 \\ &w_i\Big|_{x_i = 0} = 0, \quad \frac{\partial^2 w_i}{\partial x_i^2}\Big|_{x_i = 0} = 0 \qquad i = 1, 2 \end{aligned} \tag{A2}$$

$$\begin{aligned} &w_1\Big|_{x_1 = L_1} = w_2\Big|_{x_2 = L_2}, \qquad \frac{\partial w_1}{\partial x_1}\Big|_{x_1 = L_1} = -\frac{\partial w_2}{\partial x_2}\Big|_{x_2 = L_2} \\ &M_{x_1}\Big|_{x_1 = L_1} - M_{x_2}\Big|_{x_2 = L_2} = m, \qquad Q_{x_1}\Big|_{x_1 = L_1} = -Q_{x_2}\Big|_{x_2 = L_2} \end{aligned} \tag{A3}$$

where m is the distributed bending moment caused by the misalignment.

Due to the simply supported conditions at $y = \pm b/2$, Levy's method can be employed. Assuming that:

$$w_i(x_i, y_i) = w_i^*(x_i)\cos\frac{\pi y_i}{b} \quad (i = 1, 2), \tag{A4}$$

and introducing Eq. (A4) into Eq. (A1), one can obtain:

$$\frac{d^4 w_i^*}{dx_i^4} + \left(\frac{N_x}{D} - \frac{2\pi^2}{b^2}\right)\frac{d^2 w_i^*}{dx_i^2} + \frac{\pi^4}{b^4} w_i^* = 0 \quad (i = 1, 2). \tag{A5}$$

It can be very easily seen that, as b approaches infinity, Eq. (A5) is degenerated into the column equation, Eq. (2).

The format of the homogeneous soution of the above differential equations depends on the level of applied load. For this analysis, we are interested in the pre-buckling behavior, so we confine our discussion to the range:

$$\frac{N_x}{D} < \frac{4\pi^2}{b^2}. \tag{A6}$$

Then the characteristic equation of the differential Eq. (A5) has four complex roots, $\pm\,\epsilon \pm \eta i$. Considering the simply supported condition at $x_i = 0$, the solution of Eq. (A5) can be written as:

$$w_i^* = A_i \cosh(\epsilon x_i) \sin(\eta x_i) + B_i \sinh(\epsilon x_i) \cos(\eta x_i), \tag{A7}$$

where

$$\epsilon = \sqrt{\frac{\pi^2}{b^2} - \frac{N_x}{4D}}, \quad \eta = \frac{1}{2}\sqrt{\frac{N_x}{D}}. \tag{A8}$$

Let us assume:

$$m = m^* \cos\frac{\pi y_i}{b}. \tag{A9}$$

m should satisfy the following condition:

$$\int_{-b/2}^{b/2} m \, \mathrm{d}y = N_x e b. \tag{A10}$$

Then we can obtain:

$$m^* = \frac{\pi}{2} N_x e. \tag{A11}$$

In Eq. (A7), A_1, B_1, A_2 and B_2 are parameters to be determined. The relations between bending moment and reflection, and between shear force and deflection, are well known. By substituting these relations together with Eq. (A9), (A11) into the boundary and continuity conditions Eq. (A2), (A3), we can obtain four equations by which the constants A_1, B_1, A_2 and B_2 in Eq. (A7) can be determined. If we introduce the non-dimensional parameters of Eq. (7) then the four equations can be expressed as:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} \\ C_{21} & C_{22} & C_{23} & C_{24} \\ C_{31} & C_{32} & C_{33} & C_{34} \\ C_{41} & C_{42} & C_{43} & C_{44} \end{bmatrix} \begin{Bmatrix} A_1 \\ B_1 \\ A_2 \\ B_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 2\pi\phi_x e \\ 0 \end{Bmatrix}, \tag{A12}$$

where C_{ij} are given as follows:

$$C_{11} = T_1, \quad C_{12} = T_2, \quad C_{13} = -T_5, \quad C_{14} = -T_6, \quad C_{21} = c_0 T_4$$

$$+ s_0T_3, \quad C_{22} = c_0T_3 - s_0T_4, \quad C_{23} = c_0T_8 + s_0T_7, \quad C_{24} = c_0T_7 - s_0T_8, \quad C_{31} = (c_0^2 - s_0^2)T_1 + 2c_0s_0T_2, \quad C_{32} = (c_0^2 - s_0^2)T_2 - 2c_0s_0T_1, \quad C_{33} = -(c_0^2 - s_0^2)T_5 - 2c_0s_0T_6, \quad C_{34} = -(c_0^2 - s_0^2)T_6 + 2c_0s_0T_5, \quad C_{41} = (c_0^2 - 3s_0^2)c_0T_4 - (s_0^2 - 3c_0^2)s_0T_3, \quad C_{42} = (s_0^2 - 3c_0^2)s_0T_4 + (c_0^2 - 3s_0^2)c_0T_3, \quad C_{43} = (c_0^2 - 3s_0^2)c_0T_8 - (s_0^2 - 3c_0^2)s_0T_7, \quad C_{44} = (s_0^2 - 3c_0^2)s_0T_8 + (c_0^2 - 3s_0^2)c_0T_7. \tag{A13}$$

The constants T_i are defined as follows:

$$T_1 = \cosh(\epsilon_0R)\sin(\eta_0R), \quad T_2 = \sinh(\epsilon_0R)\cos(\eta_0R), \quad T_3 = \cosh(\epsilon_0R)\cos(\eta_0R), \quad T_4 = \sinh(\epsilon_0R)\sin(\eta_0R), \quad T_5 = \cosh[\epsilon_0(1 - R)]\sin[\eta_0(1 - R)], \quad T_6 = \sinh[\epsilon_0(1 - R)]\cos[\eta_0(1 - R)], \quad T_7 = \cosh[\epsilon_0(1 - R)]\cos[\eta_0(1 - R)], \quad T_8 = \sinh[\epsilon_0(1 - R)]\sin[\eta_0(1 - R)]. \tag{A14}$$

The other parameters used in Eq. (A12), (A13), (A14) are defined in Eq. (7) of Section 4. With the deflection functions given by Eq. (A7), the bending moment and the bending stresses can be easily calculated and the results are presented in Section 4.

References

[1] Guedes Soares C. Design equation for ship plate elements under uniaxial compression. J Construct Steel Res 1992;22:99–114.
[2] Faulkner D. A review of effective plating for use in the analysis of stiffened plating in bending and compression. J Ship Res 1975;19(1):1–17.
[3] Kmiecik M. A review of fabrication distortion tolerances for ship plating in the light of the compressive strength of plates. Lloyd’s Register Technical Association 1976;Paper No 6.
[4] Ueda Y, Yao T, Nakacho K, Yuan MG. Prediction of welding residual stress, deformation and ultimate strength of plate panels. In: Engineering design in welded constructions. Pergamon Press, 1992:251–9.
[5] Paik JK, Pedersen PT. A simplified method for predicting ultimate compressive strength of ship panels. Int Shipbuild Progr 1996;43(434):139–57.
[6] Bedair OK, Sherbourne AN. The elastic stability of partially-restrained plates under compression and in-plane bending. J Construct Steel Res 1995;35:339–60.
[7] Steup H. Compressed members with misaligned axes or loading eccentricities. J Construct Steel Res 1992;21:135–43.
[8] Gunn KW, McLester R. The effect of mean stress on the fatigue properties of aluminium alloy butt welded joints. Br Weld J 1960;7(3):201–7.
[9] Gurney TR. Fatigue of welded structures. 2rd ed. Cambridge University Press, 1979.
[10] Berge S, Myhre H. Fatigue strength of misaligned cruciform and butt joints. Norw Maritime Res 1977;5(1):29–39.

[11] Bokalrud T. Finite element analysis of butt weld and cruciform joints to determine the significance of internal crack-like defects. Norw Maritime Res 1978;6(3):23–9.
[12] Partanen T. Factors affecting the fatigue behaviour of misaligned transverse butt joint in stiffened plate structures. In: Engineering design in welded constructions. Pergamon Press, 1992.
[13] DNV Classification Note. Fatigue strength assessment of ship structures, 1993.
[14] ABS rules for building and classing steel vessels, part 5. Specialized vessels and services, 1996.