ELSEVIER

Marine Structures 11 (1998) 251–269

MARINE STRUCTURES

Effects of welding distortions and residual stresses on the ultimate strength of long rectangular plates under uniaxial compression

Weicheng Cui[a,]*, Alaa E. Mansour[b]

[a] *China Ship Scientific Research Center, P.O. Box 116, Wuxi, Jiangsu 214082, People's Republic of China*
[b] *Department of Naval Architecture and Offshore Engineering, University of California at Berkeley, Berkeley, CA 94720, USA*

Received 19 May 1997; revised version received 9 July 1998; accepted 6 October 1998

Abstract

The ultimate compressive strength of unstiffened plates is very important from the design and safety viewpoint. However, the ultimate compressive strength of these panels will depend quite significantly on the initial welding distortions and residual stresses. Currently, most of the researches concerning the effect of welding distortions concentrate only on the maximum initial distortion amplitude. However, many evidences indicate that the welding distortion shape could also affect the ultimate compressive strength significantly. In this paper, we adopt a combination of the elastic large deflection theory and the rigid-plastic analysis, proposed by Paik and Pedersen and later was generalized by the present authors. Various factors including the initial deflection shape which affect the ultimate compressive strength of unstiffened plates are investigated. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Unstiffened plates; Ultimate compressive strength; Elastic large deflection analysis; Rigid–plastic analysis; Initial welding distortion shape; Residual stresses

Nomenclature

a	plate length
A_{0i}	component of initial deflection function
A_j	component of total deflection function
b	plate width
b_t	breadth of tensile residual stress

* Corresponding author.

PII: S0951–8339(98)00012–4

D	$Et^3/12(1-v^2)$
E	Young's modulus
F	Airy's stress function
M	number of half-waves in the plate length direction for initial deflection
N	maximum number of half-waves tried for determining the final total deflection
m_{up}	number of half-waves of collapse mode for perfect plate
m_u	number of half-waves of collapse mode for imperfect plate
R_d	Strength reduction factor due to initial deflection only
R_r	Strength reduction factor due to residual stress only
t	plate thickness
w	total deflection function
w_0	initial deflection function
$w_{0\max}$	maximum value of initial deflection function
α	aspect ratio ($=a/b$)
β	plate slenderness $= b/t\sqrt{\sigma_0/E}$
v	Poisson's ratio
η	normalized residual stress ($= -\sigma_{rc}/\sigma_0$)
ϕ_{up}	normalized ultimate strength of perfect plate ($=\sigma_{up}/\sigma_0$)
ϕ_u	normalized ultimate strength of imperfect plate ($=\sigma_u/\sigma_0$)
ϕ_x	$-\sigma_{xav}/\sigma_0$ (positive value for compression)
ψ_{0i}	normalized value of A_{0i} ($=A_{0i}/t$)
$\psi_{0\max}$	normalized maximum value of initial deflection function ($=w_{0\max}/t$)
ψ_j	normalized value of $A_j(=A_j/t)$
σ_0	yield stress
σ_{rc}	compressive residual stress (as negative value)
σ_{rt}	tensile residual stress ($=\sigma_0$, as positive value)
σ_{up}	ultimate strengths of perfect plate
σ_u	ultimate strengths of imperfect plate
σ_{xav}	average axial stress in x direction (negative value for compression)

1. Introduction

Unstiffened plates are the main structural components in ships and many other structures. The ultimate strength of these elements is very important from the design and safety viewpoint because the collapse loads of these elements can often act as an indicator of the ultimate strength of the whole stiffened panel [1]. The problem has been addressed for several decades with regard to ship structures [2,3]. However, up to now, the solution to the problem is still not very satisfactory. Because of the complexity of the problem, the most frequent answer to the method for determining accurately the ultimate strength of plates is the use of finite element method (FEM). FEM is very time consuming and this is not favored by designers and practical engineers. Furthermore, although theoretically speaking, FEM could always provide a solution of desired accuracy, ironically in practice, the results obtained from FEM by different people are very diverse. Currently, there is no universal method for analysts to prove whether one particular FEM result is correct. Therefore, empirical design formulas are still preferred by the designers and vast amount of literature are available [4,5]. Faulkner has made an early attempt towards this target [6] and many others follow [7–13]. However, most of these empirical formulas are curve fitting in nature and their accuracy and applicability depends on the databases used. Ideally, the method should have some theoretical basis [5]. Mansour [3] proposed design curves that include the effect of initial deformation using a generalized Marguerr's equations (5) and (6) to include orthotropic material properties. However, with this method, a set of nonlinear equations needs to be solved. Furthermore, with the application of the elastic large deflection theory only, the ultimate strength was not defined.

Most of the researches concerning the effects of welding distortions concentrate only on the amplitude. However, there are many evidences indicating that the welding distortion shape could also affect the ultimate strength significantly [6–24]. Furthermore, it is reported that instead of the weakening effect, some initial deflection shape may have strengthening effect on the ultimate strength [7]. However, the researches dedicated to this problem are not adequate [6–24].

The purpose of this paper is to study the effects of welding distortions and residual stresses on the ultimate compressive strength of unstiffened plates. In particular, the effect of the initial deflection shape are extensively discussed.

2. Basic theory

The problem studied is a simply supported unstiffened plates subjected to uniaxial compression, see Fig. 1. The basic approach adopted in this paper for studying these effects is the one recently proposed by Paik and Pedersen [14] and generalized by the present authors [15].

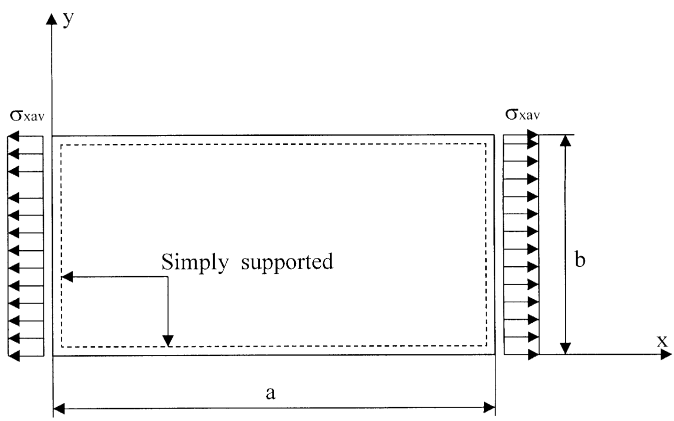

Fig. 1. A simply supported rectangular plate subjected to uniaxial compression.

2.1. Basic assumptions

The assumptions made in this development are:

(1) The initial deflection configuration can be approximated by the following Fourier Series function [14]:

$$w_0 = \sum_{i=1}^{M} A_{0i} \sin \frac{i\pi x}{a} \sin \frac{\pi y}{b} \tag{1}$$

where the number M will be selected depending on the complexity of the initial deflection shape. In this paper, $M = 11$.

(2) The elastic large deflection analysis and the rigid-plastic deflection analysis are performed individually for each component of the initial deflection function, $i = 1, 2, \ldots M$. The interaction effects between deflection components are neglected.

(3) The initial deflection with only one component from Eq. (1) is given as

$$w_0 = A_{0i} \sin \frac{i\pi x}{a} \sin \frac{\pi y}{b}, \tag{2}$$

and the possible total deflection corresponding to the above initial deflection is assumed to take the following form [15]:

$$w = A_j \sin \frac{j\pi x}{a} \sin \frac{\pi y}{b} \tag{3}$$

where j is an integer to be determined according to Assumption (6), j is not necessarily the same as i.

(4) The distribution of the residual stresses along the plate width is idealized as shown in Fig. 2:

$$\sigma_r = \begin{cases} \sigma_{rt} \ (= \sigma_0) & \text{for } 0 \leqslant y \leqslant b_t \\ \sigma_{rc} \ (\text{negative}) & \text{for } b_t < y \leqslant b - b_t \\ \sigma_{rt} \ (= \sigma_0) & \text{for } b - b_t < y \leqslant b \end{cases} \tag{4}$$

(5) For the rigid-plastic deflection analysis, only three modes of possible collapse mechanisms (modes I, II and III [14]) are considered.

(6) For each initial deflection component i, j is a certain value to be determined by varying j from 1 to N and the one which has the lowest intersection value between elastic large deflection solution and the rigid-plastic solution (see Fig. 3) for the applied stress is chosen. N is subjectively chosen. Normally the value of $N = 11$ is also adequate for practical application. Then the ultimate strength of the plate is determined as the minimum intersection value among those estimated for individual components of the initial deflection function.

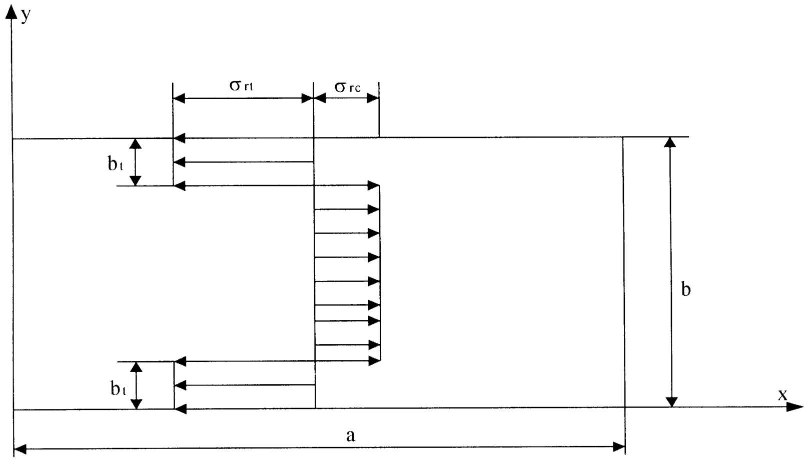

Fig. 2. Idealized welding-induced residual stress distribution.

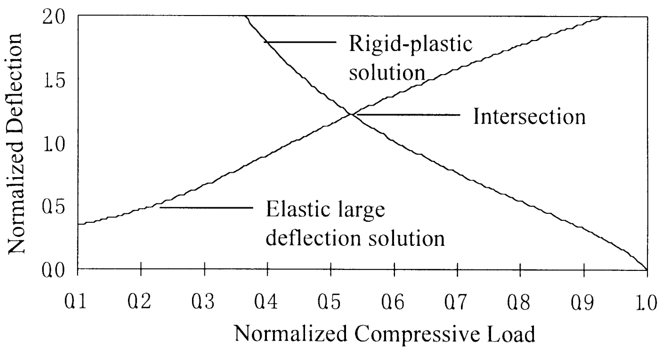

Fig. 3. Combination of the elastic and the rigid plastic responses for determination of the ultimate compressive strength.

2.2. Elastic large deflection analysis

The most general differential equations in which the initial deflection w_0 was also included are given as follows [3,14]:

$$\nabla^4 F = E[w_{,xy}^2 - w_{,xx}.w_{,yy} - w_{0,xy}^2 + w_{0,xx}.w_{0,yy}] \tag{5}$$

$$D\nabla^4(w - w_0) = t[F_{,yy}.w_{,xx} + F_{,xx}.w_{,yy} - 2F_{,xy}.w_{,xy}]. \tag{6}$$

Introducing Eqs. (2) and (3) into Eq. (5), one can obtain Airy's stress function F. Applying the Galerkin's method,

$$\int_0^a \int_0^b [D\nabla^4(w - w_0) - t(F_{,yy}.w_{,xx} + F_{,xx}.w_{,yy} - 2F_{,xy}.w_{,xy})]$$

$$\times \sin\frac{j\pi x}{a}\sin\frac{\pi y}{b}\,\mathrm{d}x\,\mathrm{d}y = 0. \tag{7}$$

Substitution of Eqs. (2)–(4) and the expression for Airy's stress function F into Eq. (7) and through integration, a cubic equation for A_j can be obtained. If we introduce the following non-dimensional parameters:

$$\psi_j = \frac{A_j}{t},\quad \psi_{0i} = \frac{A_{0i}}{t},\quad \alpha = \frac{a}{b},\quad \beta = \frac{b}{t}\sqrt{\frac{\sigma_0}{E}},\quad \eta = \frac{-\sigma_{rc}}{\sigma_0},\quad \phi_x = \frac{-\sigma_{xav}}{\sigma_0} \tag{8}$$

then the cubic equation can be written in the following form:

$$\psi_j^3 + P.\psi_j + Q = 0 \tag{9}$$

where

$$\begin{cases} P = \dfrac{4}{3(1-v^2)}\dfrac{(j^2+\alpha^2)^2}{(\alpha^4+j^4)} - \dfrac{16\beta^2}{\pi^2}\dfrac{\alpha^2.j^2}{(\alpha^4+j^4)}\left(\phi_x + \dfrac{1+\eta}{\pi}\sin\dfrac{\pi\eta}{1+\eta}\right) - \psi_{0i}^2.\dfrac{i^2.j^2+\alpha^4.\delta_{ji}}{(\alpha^4+j^4)} \\ Q = -\dfrac{4}{3(1-v^2)}\dfrac{(i^2+\alpha^2)^2}{(j^4+\alpha^4)}\psi_{0i}\delta_{ji} \end{cases} \tag{10}$$

where δ_{ji} is defined as follows:

$$\delta_{ji} = \begin{cases} 1 & j = i \\ 0 & j \neq i. \end{cases} \tag{11}$$

The detailed derivation of Eq. (9) and its solution are given in Ref. [15]. When the initial deflection and residual stresses are zero, then Eq. (9) has positive root only when $P < 0$. From this condition, it can be derived:

$$\beta \geqslant \frac{\pi}{\sqrt{3(1-v^2)}} = k_u = 1.90 \tag{12}$$

where k_u is the von Karman constant. This means that for a perfect plate, i.e. without any initial deflection and residual stresses, the ultimate strength ϕ_{up} only starts to decrease when $\beta > k_u$. Furthermore, for a given β, from $P = 0$ we can derive the same critical buckling strength formula as that from elastic small deflection theory

$$\phi_{xcr} = \frac{\pi^2}{12(1-v^2)\beta^2}\cdot\frac{(j^2+\alpha^2)^2}{j^2.\alpha^2}. \tag{13}$$

2.3. Rigid–plastic solution

Assuming a possible collapse mechanism and by equating the internal energy to the external work, one can derive a rigid-plastic solution. This solution is given in Ref. [14] for the present problem. We rewrite them in terms of the non-dimensional parameters:

$$\psi_j = \begin{cases} \dfrac{1-\phi_x^2}{\phi_x}\cdot\left(\dfrac{4}{\sqrt{16-15.\phi_x^2}}+\dfrac{\alpha/j-1}{\sqrt{4-3.\phi_x^2}}\right) & \text{for } \alpha > j \\ \dfrac{1-\phi_x^2}{\phi_x}\cdot\dfrac{4}{\sqrt{16-15.\phi_x^2}} & \text{for } \alpha = j \\ \dfrac{\alpha}{2j-\alpha}\cdot\dfrac{1-\phi_x^2}{\phi_x}\left(\dfrac{4}{\sqrt{16-15.\phi_x^2}}+\dfrac{j}{2\alpha}-\dfrac{1}{2}\right) & \text{for } \alpha < j \end{cases} \tag{14}$$

2.4. Determination of the ultimate strength

The procedure to determine the ultimate strength of the unstiffened plates is described under Assumption (6) above. Based on this procedure, a computer program is written in Fortran language. The basic input data to this program is α, β, η, ψ_{0i} ($i = 1, 2, \ldots, M$). Using this program, the ultimate strength of an unstiffened plate with a combination of any kind of initial deflection and any level of residual stresses can be calculated. In general, the ultimate strength ϕ_u and the final failure shape m_u of an imperfect plate will be a function of α, β, η, i, and ψ_{0i}. That is,

$$\begin{aligned} \phi_u &= \phi_u(\alpha, \beta, \eta, i, \psi_{0i}) \\ m_u &= m_u(\alpha, \beta, \eta, i, \psi_{0i}). \end{aligned} \tag{15}$$

In this paper, the following interesting problems are investigated based on the parametric study using this program:

(1) Effect of α and β on ϕ_{up} and m_{up}.
(2) Effect of the initial deflection shape i on ϕ_u and m_u.
(3) Effect of the initial deflection amplitude ψ_0 on ϕ_u and m_u.
(4) Effect of the residual stresses η on ϕ_u and m_u.

These problems will be addressed in the next section.

3. Discussion on various factors affecting the ultimate strength

3.1. Effect of α and β on the ultimate strength of perfect plates ϕ_{up} and m_{up}

Before we discuss the effects of welding distortions and residual stresses on the ultimate strength, it would be quite interesting to know the ultimate strength of a perfect plate. By perfect plate we mean that both residual stresses and initial deflection are zero. Therefore, the only parameters which might affect the ultimate strength are α and β. The results are shown in Fig. 4. It can be seen that α has negligible effect on ϕ_{up} and therefore only β is the significant parameter for defining the ultimate strength of an unstiffened perfect plate. This is consistent with the current literature [6].

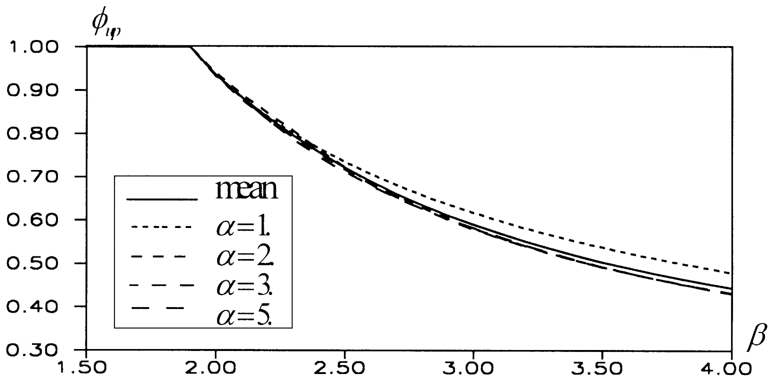

Fig. 4. Effect of α and β on ϕ_{up}.

The least-squares method is used to correlate the mean ultimate strength as a function of β and the result is

$$\phi_{up} = \begin{cases} 1 & \text{if } \beta \leqslant 1.9 \\ 0.08 + \dfrac{1.09}{\beta} + \dfrac{1.26}{\beta^2} & \text{if } \beta > 1.9. \end{cases} \tag{16}$$

As far as this type of formula is concerned, many can be found in the literature. Table 1 presents some of them.

Fig. 5 is a comparison between the present calculation and some of the existing formulas. It should be pointed out that most of the formulas in Table 1 allow some degree of initial distortions and residual stresses. Only Carlsen [19], Guedes Soares [1] and Ueda [12] have presented the empirical formulas for ϕ_u based on the experimental data in which ψ_0 and η are explicitly treated. If we set them to be zero, then ϕ_{up} can be obtained from these formulas. Only these results are plotted in Fig. 5. It can be seen that the agreement with the experimentally based formulas is generally good.

In contrast to the study on ultimate strength, the ultimate failure shape received little attention [7,12,14]. With the current method, this can also be determined. From a comprehensive parametric study, it was found that $m_{up} = 1$ when $\beta \leqslant 1.9$ and α and β have no effect. However, when $\beta > 1.9$, both α and β have significant effect on m_{up} and the results are shown in Fig. 6. The first figure shows for a given value of $\beta = 2$, how m_{up} changes with α. The second figure shows for fixed values of α, how m_{up} changes with β. It can be seen from this figure that for some values of α, m_{up} jumps once for the range $2 \leqslant \beta \leqslant 4$ while for other values of α, m_{up} jumps twice. The third figure shows the value of β at which the first jump occurs while the fourth figure shows the value of β at which the second jump occurs.

3.2. *Effect of initial deflection shape i and amplitude ψ_0 on ϕ_u and m_u*

Let us first study the typical case of $\alpha = 3$, $\beta = 2.5$. For this case $m_{up} = 4$. The results are shown in Fig. 7. The first figure shows the effect of initial deflection shape

Table 1
A summary of the formulas for the ultimate strength of unstiffened plates

Proposer	Formulas
Von Karman (1932)	$\phi_{up} = \begin{cases} 1 & \text{if } \beta \leqslant 1.9 \\ \dfrac{1.9}{\beta} & \text{if } \beta > 1.9 \end{cases}$
Winter (1940)	$\phi_u = \begin{cases} 1 & \text{if } \beta \leqslant 1.27 \\ \dfrac{1.9}{\beta} - \dfrac{0.8}{\beta^2} & \text{if } \beta > 1.9 \end{cases}$
Frankland (1940)	$\phi_u = \begin{cases} 1 & \text{if } \beta \leqslant 1.0 \\ \dfrac{2.25}{\beta} - \dfrac{1.25}{\beta^2} & \text{if } 1 < \beta \leqslant 3.5 \\ \dfrac{1.9}{\beta} & \text{if} \beta > 3.5 \end{cases}$
Gerand (1957)	$\phi_u = \begin{cases} 1 & \text{if } \beta \leqslant 1.51 \\ \dfrac{1.42}{\beta^{0.85}} & \text{if } \beta > 1.51 \end{cases}$
Faulkner (1975)	$\phi_u = \begin{cases} 1 & \text{if } \beta \leqslant 1.0 \\ \dfrac{2}{\beta} - \dfrac{1}{\beta^2} & \text{if } \beta > 1.0 \end{cases}$
Carlsen (1977)	$\phi_u = \left(\dfrac{2.1}{\beta} - \dfrac{0.9}{\beta^2}\right)\left(1 - \dfrac{0.75\psi_0}{\beta}\right)(1-\eta)$
Hughes (1983)	$\xi = 1 + \dfrac{2.75}{\beta^2}$ $\phi_u = \dfrac{1}{4}\left(1.6 + \xi - \sqrt{\xi^2 - \dfrac{10.4}{\beta^2}}\right)$
DNV (1987)	$\phi_u = \begin{cases} 1 & \text{if } \beta \leqslant 1.0 \\ \dfrac{1.8}{\beta} - \dfrac{0.8}{\beta^2} & \text{if } \beta > 1.0 \end{cases}$
Smith (1988)	$\phi_u = 0.23 + \dfrac{1.16}{\beta} - \dfrac{0.48}{\beta^2} + \dfrac{0.09}{\beta^3}$
Ueda (1992)	(a) $\eta = 0$ (i) $0.8 \leqslant \beta \leqslant 2.0$ $\phi_u = (-2.431\,\psi_0^2 + 1.6826\,\psi_0 - 0.2961)(\beta^2 - 4.0) + (7.2745\,\psi_0^2 - 4.7431\,\psi_0 + 0.6709)(\beta - 2.0) + z_1$

Table 1 (continued)

Proposer	Formulas
	$z_1 = (-0.3597\psi_0^2 + 0.1748\psi_0 + 0.8598)/(2.2432\psi_0 + 1.3322) + 0.0373\psi_0 + 0.2481$
	(ii) $2.0 \leqslant \beta \leqslant 3.5$
	$\phi_u = (-0.3597\psi_0^2 + 0.1748\psi_0 + 0.8598)/(\beta + 2.2432\psi_0 - 0.6678) + 0.0373\psi_0 + 0.2481$
Ueda (1992)	(b) $\eta = 0.11$
	(i) $0.8 \leqslant \beta \leqslant 1.6$
	$\phi_u = (-0.398\psi_0^2 + 0.4339\psi_0 - 0.1342)(\beta^2 - 2.56) + (1.0814\psi_0^2 - 0.7551\psi_0 + 0.1020)(\beta - 1.6) + z_2$
	$z_2 = (0.4974\psi_0^2 + 0.8281\psi_0 + 1.0171)/(2.7942\psi_0 + 1.2908) - 0.1849\psi_0 + 0.1571$
	(ii) $1.6 \leqslant \beta \leqslant 3.5$
	$\phi_u = (0.4974\psi_0^2 + 0.8281\psi_0 + 0.0171)/(\beta + 2.7942\psi_0 - 0.3902) - 0.1849\psi_0 + 0.1571$
Ueda (1992)	(c) $\eta = 0.25$
	(i) $0.8 \leqslant \beta \leqslant 1.5$
	$\phi_u = (-0.3317\psi_0^2 + 1.6314\psi_0 - 0.2656)(\beta^2 - 2.25) + (0.5369\psi_0^2 - 0.7798\psi_0 + 0.2854)(\beta - 1.5) + z_3$
	$z_3 = (0.292\psi_0^2 + 1.2936\psi_0 + 0.7471)/(2.897\psi_0 + 1.1189) - 1.2715\psi_0 + 0.2057$
	(ii) $1.5 \leqslant \beta \leqslant 3.5$
	$\phi_u = (0.292\psi_0^2 + 1.2936\psi_0 + 0.7471)/(\beta + 2.897\psi_0 - 0.3811) - 0.2715\psi_0 + 0.2057$
Guedes Soares (1992)	$\phi_u = (\phi_b B_b)(R_r B_r)(R_\delta B_\delta)$
	$\phi_b = \dfrac{2}{\beta} - \dfrac{1}{\beta^2}, \quad B_b = 1.08, \quad R_r = 1 - \dfrac{\Delta\phi_b}{1.08\phi_b}, \quad B_r = 1.07,$
	$R_\delta = 1 - (0.626 - 0.121\beta)\psi_0,$
	$B_{r\delta} = 0.76 + 0.148\,\eta/(1+\eta) + 0.24\,\psi_0 + 0.1\beta$
	$\Delta\phi_b = \eta\dfrac{E_t}{E_0}, \quad \dfrac{E_t}{E_0} = \begin{cases} \left(\dfrac{14.5\beta^2}{52.4+\beta^4}\right)^2 & \text{for } \beta < 2.69 \\ 1.0 & \text{for } \beta \geqslant 2.69 \end{cases}$
Present	$\phi_{u0} = \begin{cases} 1 & \text{for } \beta \leqslant 1.9 \\ 0.08 + \dfrac{1.09}{\beta} + \dfrac{1.26}{\beta^2} & \text{for } \beta > 1.9 \end{cases}$

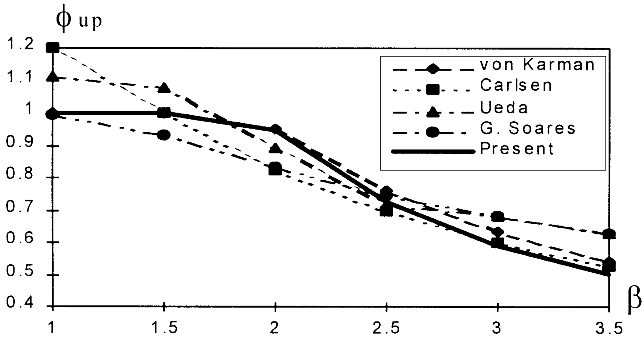

Fig. 5. A comparison of the present calculation with some existing formulas.

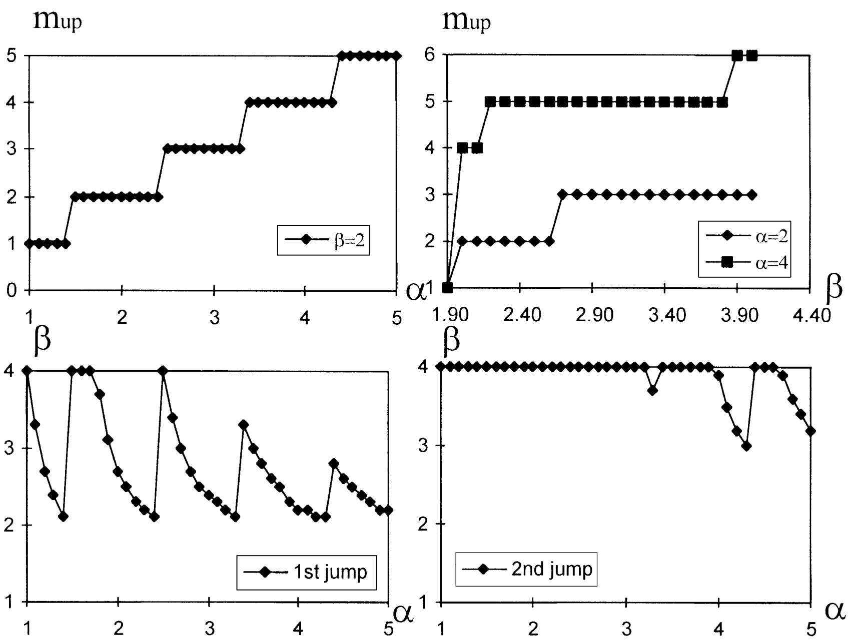

Fig. 6. Effect of α and β on the number of half-waves of collapse mode for perfect plates.

i (number of half-waves) for a given value of amplitude on the ultimate strength. It can be seen that when amplitude is small ($\psi_0 \leqslant 0.2$), then only those components close to m_{up} will have significant effect on the ultimate strength while for other components, they have small effect. As ψ_0 increases, the number of components which affect the ultimate strength significantly also increases. The second figure shows the effect of initial deflection amplitude for a given shape on the ultimate strength. It can be seen that when $i = 1$, 2 and 3, the ultimate strength will monotonically decrease with the amplitude. When i equals 4 and 5, the relation becomes a bit complicated.

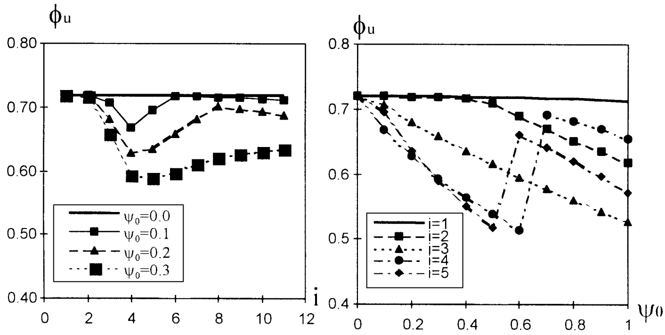

Fig. 7. Effect of initial deflection shape and amplitude on the ultimate strength for the special case of $\alpha = 3$ and $\beta = 2.5$.

It is very interesting to see that for this case if the initial deflection has only one half-wave, i.e. $i = 1$, then the initial deflection almost has no effect on the ultimate strength.

In general, it was found that if both the amplitude and the initial deflection component number are assumed to be high, the strength will decrease sharply. However, from observations of data on measured initial deflection, it can be said that as i increases A_{0i} will decrease, that is, when the initial deflection component number i is high, the corresponding amplitude will be small. Therefore, we need to establish a reasonable relation between A_{0i} and i based on practical measurements, which is studied next.

In Ref. [14], 33 cases of unstiffened plates with measured initial deflections have been given. We have plotted out a normalized initial deflection component amplitude with the component number in Fig. 8. It can be seen clearly that generally speaking, the normalized component amplitude will decrease as component number increases. Based on the mean curve for the 33 cases, using the least-squares method, the following empirical formula can be established:

$$\psi_{0i} = \psi_{0\,\max} \cdot \frac{0.756}{i^{1.565}}. \tag{17}$$

With this empirical formula, the ultimate strength can be estimated based on information of $\psi_{0\,\max}$ only. The accuracy of this formula is also checked with the 33 examples. The results are shown in Fig. 9a.

Fig. 9b and c also show the results of a correlation study based on the 33 cases. It can be stated that the correlation between $\psi_{0\,\max}$ and ϕ_u is poor while the correlation between $\psi_{0\,(mu)}$ and ϕ_u is good. This indicates that the classical use of $\psi_{0\,\max}$ without any consideration of the deflection shape such as Ref. [1,19,21] may be conservative and this deficiency can be overcome by using the empirical formula (17) established in this paper although the same information is used.

Based on Eq. (17), we used the present program to calculate the ultimate strengths of moderate level of initial deflection (Eq. (18)) and residual stresses ($\eta = 0.1$). The

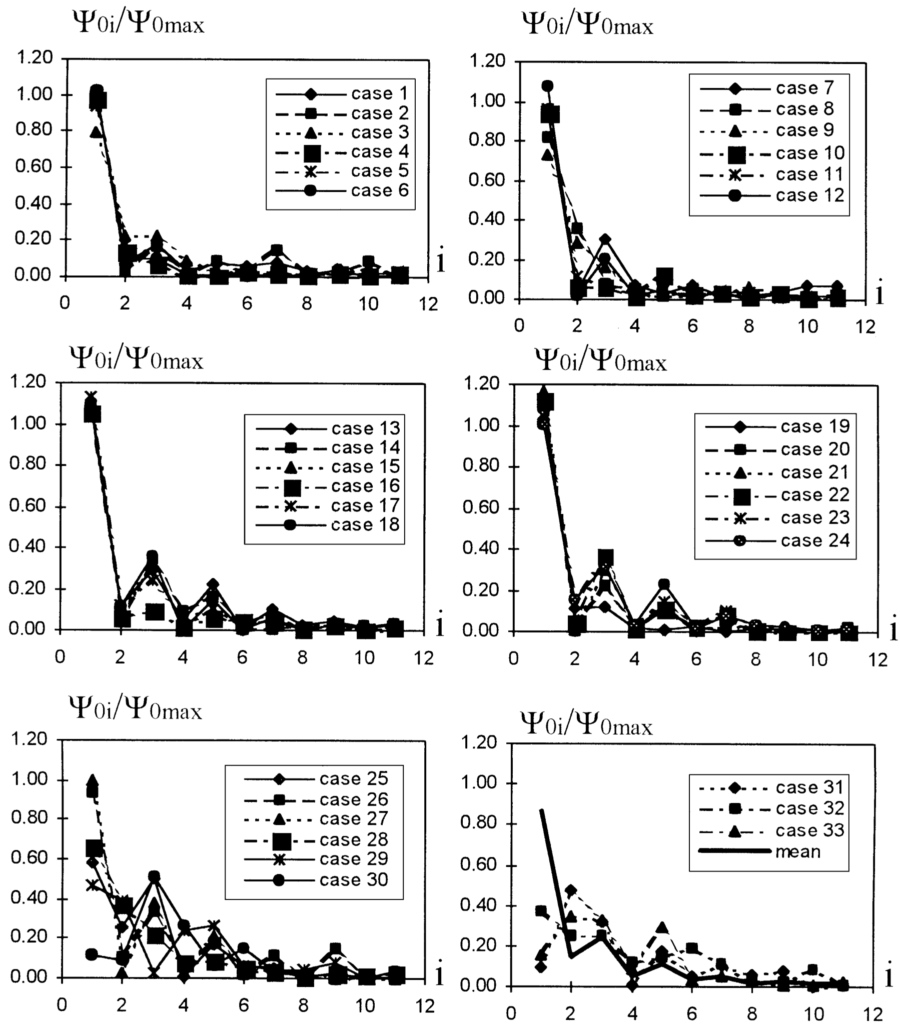

Fig. 8. Normalized initial deflection amplitude as a function of the component number.

results are compared with other empirical formulas in Fig. 10. It can be seen that the agreement is good, in particular, when $\beta > 2$.

$$\psi_{0\,\max} = \begin{cases} 0.1\beta^2 & 1 \leqslant \beta \leqslant 2.5 \\ 0.25\beta & 2.5 < \beta \leqslant 4. \end{cases} \tag{18}$$

We also used Eq. (17) to study the effect of maximum initial deflection on the ultimate strength and the results are shown in Fig. 11. It can be seen that in addition to $\psi_{0\,\max}$, both α and β have some effect on the strength reduction factor. However, in most of the current empirical formulas, the effect of α is usually ignored except Refs. [22–24].

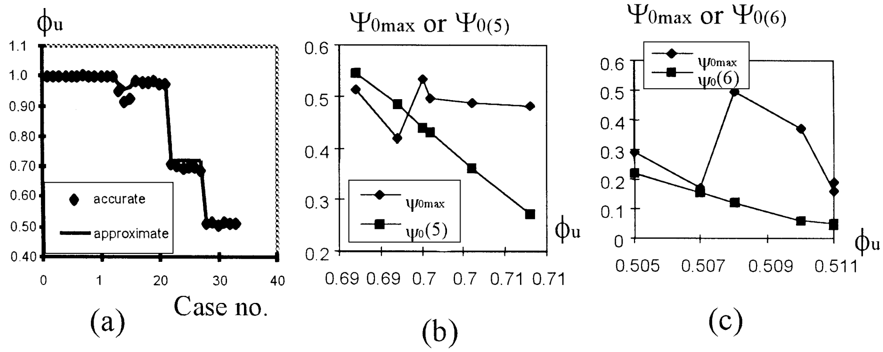

Fig. 9. Correlation of the ultimate strength ϕ_u with (a) $\psi_{0\,\max}$ with Eq. (17); (b) and (c) $\psi_{0\,\max}$ only or $\psi_0(m_u)$.

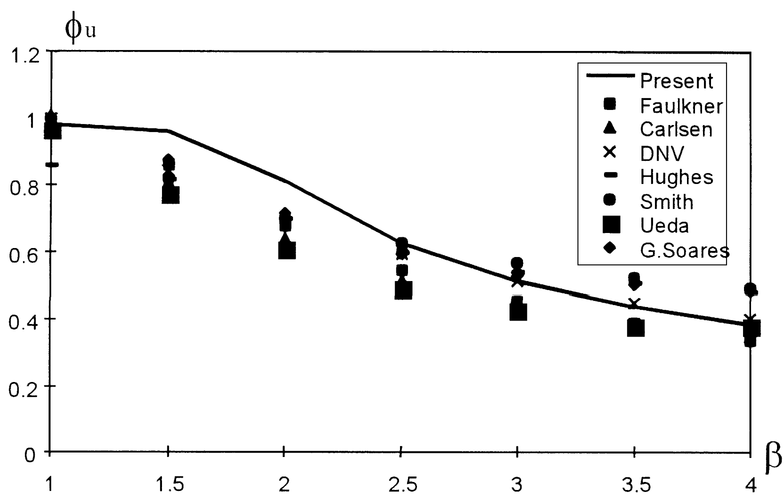

Fig. 10. A comparison of the present program with existing formulas for medium initial deflection and residual stresses.

Based on our results, the ultimate strength of a plate is presented in the following format:

$$\phi_u = \phi_{up} R_d R_r \tag{19}$$

where R_d and R_r are strength reduction factors due to initial deflection and residual stresses, respectively. Using the least-squares approach, the following formula is established for R_d based on the calculational results.

$$R_d = 1 - 0.2433\, f(\alpha) g(\beta)\, \psi_{0\,\max}^{0.911}$$

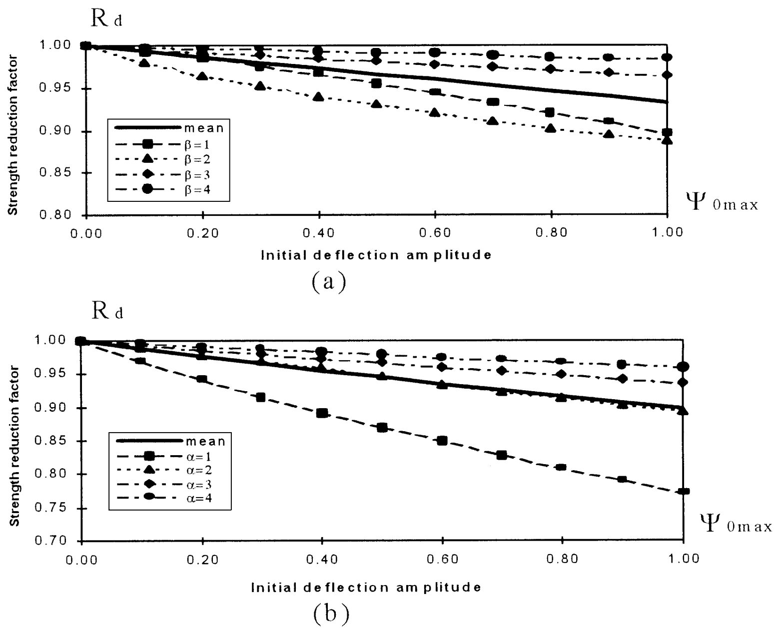

Fig. 11. Effect of α, β, and $\psi_{0\max}$ on the strength reduction factor R_d: (a) α is averaged, (b) β is averaged.

where

$$f(\alpha) = 2.05 - 1.376\alpha + 0.366\alpha^2 - 0.0345\alpha^3 \tag{20}$$

$$g(\beta) = \begin{cases} 2.28 - 2.568\beta + 1.288\beta^2 & \text{for } 1.0 \leqslant \beta \leqslant 1.9 \\ 8.191 - 4.224\beta + 0.522\beta^2 & \text{for } 1.9 < \beta \leqslant 2.5 \\ 4.593 - 2.162\beta + 0.273\beta^2 & \text{for } 2.5 < \beta \leqslant 4.0. \end{cases}$$

The goodness of the fit of Eq. (20) is shown in Fig. 12. Using Eq. (17), the final deflection shape m_u is mainly dominated by α and β, but $\psi_{0\max}$ also has some effect.

3.4. Effect of residual stresses η on ϕ_u and m_u

The effect of η on ϕ_u is shown in Fig. 13. It can be seen that α has little effect on the strength reduction factor R_r and so we average the values for α. Using the least-squares method, we can derive the following expression for R_r:

$$R_r = 1 - 0.46(\beta - 1.5)^{0.275}\eta^{0.725}. \tag{21}$$

The parameter η does have some effect on m_u but generally speaking, this is small.

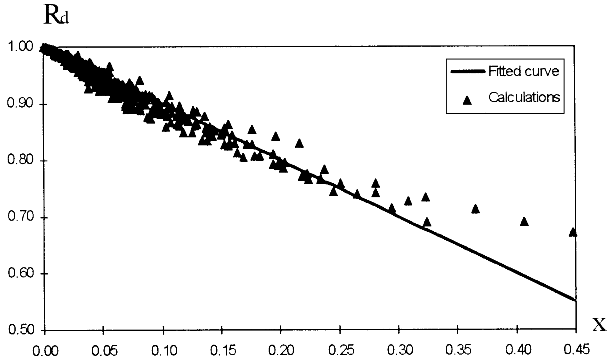

Fig. 12. Goodness of the fit of Eq. (20) to the calculational results.

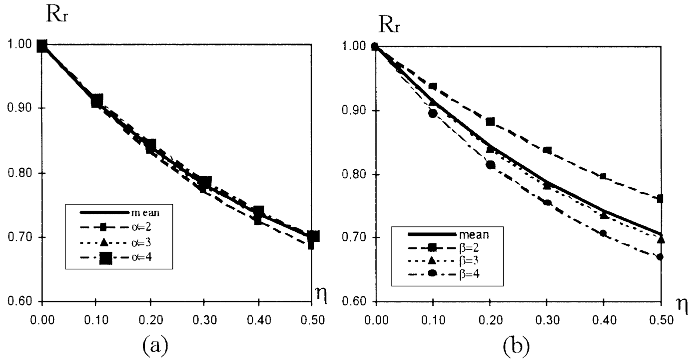

Fig. 13. Effect of α, β, and η on the strength reduction factor R_r: (a) $\beta = 2.5$; (b) α is averaged.

3.5. Application of the present results in design and analysis

The method introduced in this paper together with many calculated results can be directly applied in the design process or for analyzing actual unstiffened plates. If an unstiffened plate is given with measured residual stresses and initial deflection, then one can directly apply the present method to predict its ultimate compressive strength. The method is very efficient and it takes only seconds to run a case in a personal computer. From the comparisons with other experimentally-based empirical formulas given above, the results from the present method are quite accurate. Furthermore, if one does not want to develop a program, one can simply use the formulas, Eq. (19) together with Eqs. (16), (20) and (21) to estimate the ultimate strength. The results will also be quite accurate. If one is required to design a plate, then only the maximum value of initial deflection function is normally specified. With this value, one can use Eq. (17) to calculate the amplitude for each component and then apply the present detailed method to calculate the ultimate strength. Alternatively, he can also directly use the approximate formulas to estimate the strength based on the specified levels of maximum initial deflection and residual stresses.

4. Summary and conclusions

The ultimate compressive strength of ship panels will depend quite significantly on the initial welding distortions and residual stresses. Evidence indicates that the welding distortion shape may also affect the ultimate compressive strength significantly. In this paper, we adopt a combination of the elastic large deflection theory and the rigid-plastic analysis to study the impact of various factors including the initial deflection shape on the ultimate compressive strength of unstiffened plates. Through this investigation, the following conclusions can be drawn:

(1) The simplified method recently proposed by Paik and Pedersen [14] and generalized by the present authors [15] is a very powerful tool in predicting the ultimate compressive strength of ship panels. The results obtained from this method are well comparable to the experimental data-based empirical formulas. The advantage of this method over the curve fitting empirical formulas is that this method is based on a sound theory of mechanics, but it still retains the simplicity and possible application in design.

(2) Through a comprehensive parametric study, it is proved that for a perfect plate, the effect of the aspect ratio α on ultimate strength is negligible. However, it will quite significantly affect the final total deflection shape at collapse.

(3) Based on this method, it was confirmed that in addition to the amplitude, the initial deflection shape (i.e. the number of half waves) does have a significant effect on the ultimate strength. Generally speaking, only those initial deflection whose shape is close to the final total deflection plays a significant role. The rest of shapes of the initial deflection may have negligible or small effect on the ultimate strength.

(4) Through a detailed correlation study based on the 33 cases provided in Ref.[14], it is also confirmed that the correlation between the maximum initial deflection amplitude and the ultimate compressive strength is generally poor and only the amplitude of the initial deflection component which has the same form as the final total deflection plays a significant role in ultimate strength. This is a further evidence of the applicability of the present theory. An empirical formula is proposed for calculating the amplitude of the initial deflection component based only on information on the maximum initial deflection amplitude.

(5) Based on the proposed empirical formula, the effect of the maximum initial deflection amplitude on the ultimate strength is studied and it is found that in addition to the maximum initial deflection amplitude, both α and β have some effect on the strength reduction factor R_d. An empirical formula is proposed.

(6) However, α has negligible effect on the strength reduction factor associated with the residual stress, R_r. Based on the comprehensive parametric study, an empirical formula for R_r is also proposed.

Acknowledgements

This research was carried out during the first author's visit to University of California at Berkeley. The financial support from Chinese Scholarship Council for the visit of the first author is highly appreciated. The support of NAVSEA contract no. N00024-96-C-4123 is also acknowledged.

References

[1] Guedes Soares C. Design equation for ship plate elements under uniaxial compression. J Construct Steel Res 1992;22:99–114.
[2] Ueda Y, Tall L. Inelastic buckling of plates with residual stresses. Publications International Association of Bridge and Structural Engineer, 1967:211–53.
[3] Mansour AE. Post-buckling behaviour of stiffened plates with small initial curvature under combined loads. Int shipbuild progr vol.18, June 1971.
[4] Hsu PH, Wu YS, editors. Proc 11th Int Ship and Offshore Structures Congr. Wuxi, China, 1991.
[5] Jeffrey NE, Kendrick AM, editors. Proc 12th Int Ship and Offshore Structures Congr. St. John's Canada, 1994.
[6] Faulkner D. A review of effective plating for use in the analysis of stiffened plating in bending and compression. J Ship Res 1975;19(1):1–17.
[7] Kmiecik M. A review of fabrication distortion tolerances for ship plating in the light of the compressive strength of plates. Lloyd's register technical association. Paper No.6, 1976.
[8] Frieze PA, Dowling PJ, Hobes RH. Ultimate load behaviour of plates in compression. In: Steel plated structures. London: Crosby Lockwood Staples 1977:24–50.
[9] Harding JE, Hobes RE, Neal BG. The elasto-plastic analysis of imperfect square plates under in-plane loading. Proc Inst Civil Engrs Part 2, vol. 63. 1979:137–58.
[10] Carlsen CA, Czujko J. The specification of tolerances for post-welding distortions of stiffened plates in compression. Struct Eng 1978;56A(5).
[11] Dow RS, Smith CS. Effects of localised imperfections on the compressive strength of long rectangular plates. J Construct Steel Res 1983;3:51–76.
[12] Ueda Y, Yao T. The influence of complex initial deflection modes on the behaviour and ultimate strength of rectangular plates in compression. J Construct Steel Res 1985;5.
[13] Guedes Soares C, Kmiecik M. Simulation of the ultimate compressive strength of an unstiffened rectangular plates. Marine Struct 1993;6:553–69.
[14] Paik JK, Pedersen PT. A simplified method for predicting ultimate compressive strength of ship panels. Int Shipbuild Progr 1996;43(434):139–57.
[15] Cui WC, Mansour AE. Generalization of a simplified method for predicting ultimate compressive strength of ship panels. Int Shipbuild Progr, accepted.
[16] Murray NW. Analysis and design of stiffened plates for collapse load. Struct Eng 1975;53:153–8.
[17] Rhodes J. On the approximate prediction of elasto-plastic plate behaviour. Proc Inst Civil Engrs Part 2, 1981;65:165–83.
[18] Hughes OF. Ship structural design: a rationally-based, computer-aided optimization approach. New York: Wiley, 1983.
[19] Carlsen CA. Simplified collapse analysis of stiffened plates. Norw Marit Res 1977;7(4):2–36.
[20] Smith CS, Davidson PC, Chapman JC, Dowling PJ. Strength and stiffness of ship's plating under in-plane compression and tension. Trans RINA 1988;130.
[21] Ueda Y, Yao T, Nakacho K, Yuan MG. Prediction of welding residual stress, deformation and ultimate strength of plate panels. In: Engineering design in welded constructions. Oxford: Pergamon Press 1992:251–9.
[22] Guedes Soares C, Faulkner D. Probabilistic modeling of the effect of initial imperfections on the compressive strength of rectangular plates. Proc. 3rd Int Symp on Practical Design of Ships and Mobile Units (PRADS). Trondheim 1987;2:783–95.

[23] Guedes Soares C, Gordo JM. Collapse strength of rectangular plates under transverse compression. J Construct Steel Res 1996;36(3):215–34.

[24] Guedes Soares C, Kmiecik M. In: Graczyk T, Jastrzebski T, Brebbia CA, Burns R, editors. Influence of the boundary conditions on the collapse strength of square plates with initial imperfections, marine Technology and Transportation Conference. Computational Mechanics Publications, 1995:227–35.